Homework #9 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points): Given a magnetic field of the form $\vec{B} = kz\,\hat{x}$ (with k a constant), find the force on a square loop with sides of length a lying in the y-z plane, centered at the origin. The loop has a current I that flows counterclockwise as seen from a viewpoint looking along the x-axis.

Problem 2 (20 points): Consider a total current I flowing down a cylindrical wire with a circular cross-section of radius a.

2a (10 points): If the current I flows entirely on the surface of the wire (uniformly distributed across the surface), what is the surface current density K?

2b (10 points): If instead the volume current density is inversely proportional to the distance s from the axis, what is $J(s)$ in terms of I and a?

Problem 3 (30 points): Calculate the magnetic field at the center of a uniformly charged spherical shell of radius R, carrying total charge Q, and spinning around the z-axis with a uniform angular velocity ω. *Hint: Start with the solution derived for the magnetic field above/below the center of a circular loop of current.*

Problem 4 (30 points): Consider two infinite straight line charges with linear charge density λ, aligned parallel to each other and separated by a distance d. How fast would these two line charges have to move in order for the magnetic attraction between the wires to balance the electrostatic repulsion? Is this possible?